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() :

$$G(u,u_x,u_{xx},\dots,u_x^{(n)},u_y,u_{yy},\dots,u_y^{(m)},u_{xy}^{(i+j)},\dots)=0 \quad () :$$

$$u_x = \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} = u_y \quad u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$$

$$\begin{cases} u_{xxx}(x,y) + u_{yyy}(x,y) = 0 \\ \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} = 0 \end{cases} :$$

PDE

$$(() \quad) \quad \text{PDE}$$

. PDE A , B ,, G .

$$Au_{xx}(x,y)+Bu_{xy}+Cu_{yy}(x,y)+Du_x(x,y)+Eu_y(x,y)+Fu(x,y)+G=0:()$$

.

x , y A , B ,, G

.

U A , B ,, G

$$u_{xx}+(\sin u)_{uy}=0:$$

$$zu_{xx}(x,y)+xy^2u_{yy}(x,y)+e^xu(x,y)=10:$$

$$u(x,y,z)$$

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U

Boundary value

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U

. Initional Value Problems () Problems

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Homogenous PDE

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PDE

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. Fix L

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$$v(x+\Delta x,t)$$

y

:

x

$$H(v,t)=H(x+\Delta x,t)\rightarrow H(x,t)=cte\quad \forall t\quad 0<x<l$$

$$\tan \alpha = \frac{v(x+\Delta x,t)}{H} \qquad \tan \beta = \frac{-v(x,t)}{-H}$$

$$f=-by_t(x,t)$$

$$y_{tt}(x,t)=H/\delta y_{xx}(x,t)-by_t(x,t)$$

$$\begin{aligned} y(0,t)&=0 \\ y(l,t)&=0 \\ y(x,0)&=f(x) \quad 0<x<\ell \end{aligned}$$

$$\begin{aligned} y(0,t)&=0 \\ y(\ell,t)&=0 \\ y(x,0)&=0 \\ y_t(x,0)&=g(x) \end{aligned}$$

$$\begin{aligned} &y(x,t). \\ &X+\Delta X \\ &X+\Delta X \\ &F(X,t) \end{aligned}$$

$$F = y^{(v)}(x + \Delta x) - y(x, t) \quad ;$$

$$\quad \quad \quad ; A$$

$$F(x, t) = \frac{E}{\Delta x} (y(x + \Delta x, t) - y(x, t)) \quad (v) \quad (\quad) \quad ; B$$

$$F(x, t) = AE y_x(x, t) \quad .$$

$$F(x, t) = -E y_x(x, t) \quad ;$$

$$\quad \quad \quad x$$

$$F(x + \Delta x, t) = AE y_x(x + \Delta x, t)$$

$$\quad \quad \quad ; \quad (\quad)$$

$$F(x + \Delta x, t) - F(x, t) = \delta A \Delta x y_{xx}(x, t) \quad (\quad) \quad ; A$$

$$\Rightarrow y_{tt}(x, t) = \frac{AE}{\delta A} \left(\frac{y(x + \Delta x, t) - y(x, t)}{\Delta x} \right) \quad ; \Delta$$

$$\quad \quad \quad \delta$$

$$y_{tt}(x, t) = \frac{E}{\delta} y_{xx}(x, t) \quad ; (\quad)$$

$$Z_{tt}(x, y, t) = H / \delta (Z_{xx}(x, y, t) + Z_{yy}(x, y, t)) \quad ;$$

$$\quad \quad \quad . \quad t \quad y \quad x \quad Z_{tt}$$

$$\quad \quad \quad ;$$

$$\quad \quad \quad t \quad x, y \quad u(x, y, z, t) \quad R$$

$$\phi = -K \frac{du}{dn} \quad n \quad .$$

$$\quad \quad \quad ; n \quad ; k \quad ; \phi$$

$$\quad \quad \quad (\quad) \quad z \quad y \quad x$$

$$\phi_1 = -K \frac{du}{dx}$$

$$\left(\begin{array}{c} \end{array} \right) : y$$

$$\phi_2 = -K \frac{du}{dy}$$

$$\phi_3 = -K \frac{du}{dz}$$

.

y

$$J = \phi_1 \hat{I} + \phi_2 \hat{J} + \phi_3 \hat{K}$$

$$J = -k \left(\frac{\partial u}{\partial x} \hat{I} + \frac{\partial u}{\partial y} \hat{J} + \frac{\partial u}{\partial z} \hat{K} \right)$$

$$J = -k(\Delta u)$$

$$\phi = J.n$$

$$\phi$$

.

: yz

:

$$\phi = j \quad j = -k \frac{du}{dx}$$

:

$$I_1$$

$$I_1$$

$$I_1 = \iiint_{R_0} \delta u dv \quad \left(\begin{array}{c} \end{array} \right)$$

.

:

$$I_1 = \iint_R J . dA$$

$$I_2 = \iiint_R (div j) dv = \iiint_R \nabla (J) dV$$

:(

—

):

$$I_2 = \iiint_R -k \nabla . J dv = \iiint -k \nabla^2 u dV$$

$$I_1 = I_2 \quad : \left(\begin{array}{c} \end{array} \right)$$

$$I_1 - I_2 = 0 \Rightarrow \iiint_R (\delta(u_t) - ku(vu)) dv = 0$$

.

U

R

$$\delta u_t = ku(v_u) = 0 \Rightarrow \Delta U_t = k \nabla^2 u$$

$$\nabla u(x,y,t,z)vu = \frac{\partial u}{\partial x} i^{\wedge} + \frac{\partial u}{\partial y} j^{\wedge} + \frac{\partial u}{\partial z} k^{\wedge}$$

$$u_1=M_1\hat{i}+M_2\hat{j}+M_3\hat{k}$$

$$U_M=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\left(M_1\hat{i}+M_2\hat{j}+M_3\hat{k}\right)=\frac{\partial M_1}{\partial x}+\frac{\partial M_2}{\partial y}+\frac{\partial M_3}{\partial z}$$

$$:\quad \overline{\mu}=Uu$$

$$K\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}\right)=\delta u_t(x,y,z,t)\hspace{10em}:$$

$$\frac{\partial^2u(x,y,z,t)}{\partial x^2}+\frac{\partial^2(x,y,z,t)}{\partial y^2}+\frac{\partial^2u(x,t,z,t)}{\partial z^2}=\frac{\partial}{k}\frac{\partial u}{\partial t}(x,y,z,t)$$

$$u_{xx}+u_{yy}+u_{zz}=\frac{\delta}{k}u_t$$

$$\text{PDE}$$

$$\text{PDE}$$

$$\text{دارد E}$$

$$\text{P D E}$$

$$? \nabla^2 u = \frac{\partial}{k} u_t \qquad \nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial x^2} \qquad .$$

$$:$$

$$:$$

$$k\frac{du}{dn}|=\phi_0$$

$$\frac{du}{dx}=0\hspace{10em}:$$

$$:$$

$$k\frac{du}{dn}=h(T_0-u)$$

$$\nabla^2 u = \frac{\delta}{k} \frac{\partial u}{\partial t} \Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\delta}{k} \frac{\partial u}{\partial t}$$

$$\left. k \frac{du}{dx} \right|_{x=0} = 0 \quad u_x(x,0,t) = 0 \quad 0 < x < l$$

$$0 < x < l, \quad y = 0$$

$$\left. k \frac{du}{dx} \right|_{x=0} = \phi_0 \quad Ku_x(0,y,t) = \phi_0 \quad y > 0$$

$$\left. k \frac{du}{dx} \right|_{x=e} = h(o,u) \quad KUx(0,y,t) = -h(u) \quad y > o$$

$$u(x,y,0)=f(x)$$

$$\nabla^2 u = \partial^2 u / \partial t^2$$

$$\nabla^2 u = \delta(u,x,y,z,t)$$

$$c_1u_1 - c_2u_2 \qquad R \qquad c_2, c_1 \qquad u_2, u_1$$

$$e\{c_1u_1 + c_2u_2\} = c_1e\{u_1\} + c_2e\{u_2\} \qquad u_2 \mathcal{J} u_1$$

$$e^{-}$$

$$e\{u\} = \int u dt \rightarrow e\{c_1u_1 + c_2u_2\} = \int (c_1u_1 + c_2u_2) du =$$

$$\int c_1 u_1 du + \int c_2 u_2 du = c_1 \int u_1 du + c_2 \int u_2 du \qquad \qquad \qquad :$$

$$e\{0\}=0$$

$$e\left\{\sum_{k=1}^nc_ku_k\right\}=\sum_{k=1}^nc_ke\{u_k\}$$

$$: \hspace{10em} M$$

$$e\{M\{u\}\}=M\{e\{u\}\}=eM\{u\}$$

$$(e+M)\{u\}=e\{u\}+M\{u\}$$

$$:$$

$$A\frac{\partial^2u}{\partial x^2}+B\frac{\partial^2u}{\partial xy}+c\frac{\partial^2u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu(x,y)=G(x,y)$$

$$e \hspace{10em} G=0$$

$$:$$

$$e=A\frac{\partial^2}{\partial x^2}+B\frac{\partial^2}{\partial xy}+C\frac{\partial^2u}{\partial y^2}+D\frac{\partial}{\partial x}+E\frac{\partial}{\partial y}+F$$

$$e$$

$$e\{u\}=0 \hspace{1.5em} (\hspace{10em} u).$$

$$e\{u\}=G$$

$$e\{u_1\}=0 \hspace{1em} e\{u_2\}=0 \hspace{10em} : \hspace{10em} u_1,u_2$$

$$c_1e\{u_1\}+c_2e\{u_2\}=e\{c_1u_1+c_2u_2\}=0$$

$$\hspace{10em} u_1,u_2 \hspace{1em} :$$

$$\hspace{10em} -$$

$$\hspace{10em} u_2 \hspace{10em} u_1$$

$$e\{u_1\}=0 \hspace{1.5em} e\{u_2\}=G$$

$$\hspace{10em} u=cu_1+u_2 \hspace{1em} u$$

$$e\{cu_1+u_2\}=ce\{u_1\}+e\{u_2\}=G$$

$$u_t(x,t)=Ku_{xx}(x,t) \hspace{10em} :$$

$$\begin{aligned} u_x(o,t) &= 0 \\ u_x(c,t) &= 0 \end{aligned} \quad :$$

$$u_0(x,t) = \frac{1}{2} \quad U_n(x,t) = \exp\left(-\frac{n^2\pi^2k}{c^2}t\right) \cos\frac{n\pi x}{c} \quad n = 1, 2, \dots$$

$$u_t(x,t) = \left(\frac{-n^2\pi^2k}{c^2}\right) \exp\left(\frac{-n^2\pi^2k}{c^2}t\right) \cos\frac{n\pi x}{c} \quad t$$

$$u_{xx} = \left(\frac{-n^2\pi^2}{c^2}\right) \exp\left(\frac{-n^2\pi^2}{c^2}kt\right) \cos\frac{n\pi x}{c} \quad k$$

$$ku_{xx} = u(x,t)$$

$$u = c_o u_o + c_1 u_1 + \dots + c_n u_n$$

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$$

x , y , t F A

$$\cdot \quad \quad \quad \text{F-A}$$

$$\begin{aligned} B^2 - 4Ac = 0 & \quad \text{H yperbolic} & B^2 - 4Ac > 0 \\ \cdot (\text{Parabolic}). & & \text{Eccllipite} \end{aligned}$$

$$:$$

$$u$$

$$\cdot \quad \quad \quad u$$

$$\cdot \quad \quad \quad Ku + \frac{du}{dn} \quad \quad \quad (\quad)$$

$$u_{tt}, u_t$$

$$u_t \quad u$$

$$:$$

$$\begin{cases} x = p \cos \phi \\ y = p \sin \phi \\ z = z \end{cases} \quad \begin{cases} p = \sqrt{x^2 + y^2} \\ \phi = \text{Arc tan} \left(\frac{y}{x} \right) \\ z = z \end{cases}$$

←

$$r = \sqrt{p^2 + z^2} \quad \theta = \text{Arc tan} \left(\frac{p}{z} \right)$$

$$\phi = \phi$$

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

←

$$\begin{cases} p = r \sin \theta \\ \phi = \phi \\ z = r \cos \theta \end{cases}$$

←

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\phi = \text{ArcTan} \frac{y}{x}$$

←

$$z = \text{ArcTan} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\nabla^2 u = 0 \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad :$$

$$u(p, \phi, z) \quad u$$

:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial \phi} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x}$$

$$= \frac{\partial u}{\partial p} \frac{x}{\sqrt{x^2 + y^2}} + \frac{\partial u}{\partial \phi} \left(\frac{-y}{x^2 + y^2} \right) = \frac{\partial u}{\partial p} \cdot \frac{x}{p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} = \cos \phi \frac{\partial u}{\partial p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} *$$

$$: \quad * \quad u$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)$$

$$\frac{\partial^2 u}{\partial x^2} = \cos \phi \left(\frac{\partial}{\partial p} \left(\frac{\partial u}{\partial x} \right) \right) - \frac{\sin \phi}{p} \left(\frac{\partial}{\partial \phi} \left(\frac{\partial u}{\partial x} \right) \right)$$

$$\frac{\partial^2 u}{\partial x^2} = \cos \left(\frac{\partial}{\partial p} \left(\cos \phi \frac{\partial u}{\partial p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} \right) \right) - \frac{\sin \phi}{p} \left(\frac{\partial}{\partial \phi} \cos \phi \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} \right)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \cos \phi \left(\cos \phi \frac{\partial^2 u}{\partial p^2} + \frac{\sin \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} \right) -$$

$$\frac{\sin \phi}{p} \left(-\sin \phi \frac{\partial u}{\partial p} + \cos \phi \frac{\partial^2 u}{\partial u \partial \phi} - \frac{\cos \phi}{p} \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial^2 u}{\partial \phi^2} \right)$$

$$= \cos^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{\cos \phi \sin \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{\cos \phi \sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin^2 \phi}{p} \frac{\partial u}{\partial p} -$$

$$\frac{\sin \phi \cos \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi} + \frac{\sin^2 \phi}{p} \frac{\partial^2 u}{\partial \phi^2} \Rightarrow$$

$$\cos^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{2 \sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{2 \cos \phi \sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin^2 \phi}{p} \frac{\partial u}{\partial p} + \frac{\sin^2 \phi}{p^2} \frac{\partial^2 u}{\partial \phi^2}$$

$$\frac{\partial^2 u}{\partial y^2} :$$

$$\frac{\partial u}{\partial y} = \sin \phi \frac{\partial u}{\partial p} + \frac{\cos \phi}{p} \frac{\partial u}{\partial \phi} \Rightarrow \frac{\partial^2 u}{\partial y^2} = \sin \phi \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial y} \right) + \frac{\cos \phi}{p} \frac{\partial}{\partial \phi} \left(\frac{\partial u}{\partial y} \right)$$

$$\frac{\partial^2 u}{\partial y^2} = \sin^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{2 \sin \phi \cos \phi}{p} \frac{\partial^2 u}{\partial p \partial \phi} + \frac{\cos^2 \phi}{p^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\cos^2 \phi}{p} \frac{\partial u}{\partial p} - \frac{2 \sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 (x, y, z)}{\partial z^2} \quad \nabla^2 u = 0$$

$$\nabla^2 u = 0 \rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \Rightarrow$$

$$\frac{\partial^2 u}{\partial p^2} + \frac{1}{p} \frac{\partial u}{\partial p} + \frac{1}{p^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

:

$$P^2 \frac{\partial^2 u}{\partial p^2} + p \frac{\partial u}{\partial p} + \frac{\partial^2 p}{\partial \phi^2} p^2 \frac{\partial^2 u}{\partial z^2} = 0$$

: Z

$$P^2 \frac{\partial^2 u}{\partial p^2} + p \frac{\partial u}{\partial p} + \frac{\partial^2 p}{\partial \phi^2} p^2 \frac{\partial^2 u}{\partial z^2} = 0 \quad p \frac{\partial^2 u}{\partial z^2} = 0$$

:

$$x^2 \frac{\partial u}{\partial x^{2+}} + x \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \begin{matrix} 0 < x < \infty \\ 0 < y < 2\pi \end{matrix}$$

(). :

$$\nabla^2 u = 0 \Rightarrow \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\cos \theta}{r^2} \frac{\partial u}{\partial \theta} = 0 \quad :$$

:

$$\nabla^2 u = \frac{1}{r} (ru)_{rr} + \frac{1}{r^2 \sin^2 \theta} u_{\phi\phi} + \frac{1}{r^2 \sin \theta} (u_{\theta} \sin \theta)_{\theta}$$

$$(u + ru_r) \rightarrow (u_r + u_r + ru_{rr})$$

:

$$\nabla^2 u = \frac{1}{r^2} (r^2 u_r)_r + \frac{1}{r^2 \sin^2 \theta} u_{\phi\phi} + \frac{1}{r^2 \sin \theta} (u_{\theta} \sin \theta)_{\theta} \quad : \quad \phi$$

$$r \frac{1}{r^2} (r^2 u_r)_r + \frac{1}{r^2 \sin \theta} (u_{\theta} \sin \theta)_{\theta} = \nabla^2 u$$

$$\nabla^2 u = G \quad :$$

(Sepration .

of Vauiable)

• •

$$\left\{ \begin{array}{l} Ku_{xx}(x,t) = u_t(x,t) \\ u_x(0,t) = 0 \\ u_x(c,t) = 0 \\ u(x,0) = f(x) \end{array} \right.$$

$$f(\mathbf{x})$$

u

$$u(x, t)$$

•

$$1 \quad X'(o)T(t) = X'(c)T(t) = 0$$

$$\Rightarrow T(t) \neq 0, X'(o) = o_1 X'(c) = 0$$

$$\begin{cases} X'(x) - \lambda x(x) = 0 \\ X(o) = 0 \\ X'(c) = 0 \end{cases} \quad T'(t) - k\lambda t(t) = 0 \Rightarrow \frac{T'(t)}{T(t)} = k\lambda \Rightarrow \cos T(t) = k\lambda t + \cos A$$

$$\Rightarrow \cos \frac{T(t)}{A} = k\lambda t \Rightarrow \frac{T(t)}{A} = e^{k\lambda t} \Rightarrow T(t) = Ae^{k\lambda t}$$

: _

$$1) \lambda > 0 \quad X''(x) - \lambda X(x) = 0 \Rightarrow X(x) = a_1 e^{\sqrt{\lambda}x} + a_2 e^{-\sqrt{\lambda}x} \quad \lambda > 0$$

$$\text{فرض} \quad \lambda = a^2 \Rightarrow X''(x) = a^2 \times (x) \Rightarrow X(x) = Ae^{ax} + Be^{-ax}$$

$$X(x) = Aae^{ax} - Bae^{-ax}$$

$$\begin{cases} X'(o) = 0 \Rightarrow A - B = 0 \\ X'(c) = 0 \Rightarrow Ae^{ac} - Be^{-ac} = 0 \end{cases} \Rightarrow A = 0, B = 0$$

$$\cdot \quad \lambda > 0 \quad u(x, t) \quad X(x)$$

$$2) \lambda = 0 \quad X''(x) = 0 \Rightarrow X(x) = Ax + B$$

$$X'(o) = 0 \Rightarrow A = 0$$

$$X'(c) = 0 \Rightarrow A = 0$$

$$X(x) = B \quad T(t) = A \quad u(x, t) = X(x)T(t) - A.B$$

$$u(x, o) = f(x) \Rightarrow AB = f(x)$$

$$\cdot \quad u(x, t) = f \quad f(x)$$

$$3) \lambda < 0 \quad X''(x) = -a^2 X(x) \Rightarrow X(x) = A \cos ax + B \sin ax$$

$$\lambda = -a^2$$

$$X'(o) = 0 \Rightarrow -Aa \sin ac + aB \cos ax = 0 \Big|_{x=0} \quad B = 0$$

$$X'(o) = 0 \Rightarrow -Aa \sin ac = 0 \Rightarrow \sin ac = 0 \rightarrow ac = n\pi \Rightarrow a = \frac{n\pi}{c}$$

$$X(x) = \cos\left(\frac{n\pi}{c}x\right) \quad n = 0, 1, \dots$$

$$T(t) = e^{k\lambda t} = e^{-ka^2 t} = e^{-k\left(\frac{n^2\pi^2}{c^2}\right)t} \quad n = 0, 1, \dots$$

$$\cdot \quad \quad \quad ku_{xx} = u_t \quad \quad \quad u_n$$

$$u = \sum_{n=1}^{\infty} a_n u_n(x, t)$$

$$u(x, t) = X(x)T(t) = e^{-k\frac{n^2\pi^2}{c^2}t} + \cos\frac{n\pi x}{c}$$

$$u_n(x, t) = e^{-\frac{kn^2\pi^2}{c^2}t} \cos\left(\frac{n\pi x}{c}\right) \quad n = 0, 1, \dots$$

$$u(x, t) = \sum_{n=1}^{\infty} a_n e^{-\frac{n^2\pi^2}{c^2}kt} \cos\frac{n\pi x}{c}$$

$$\vdots \quad \quad \quad a_n$$

$$u(x, 0) = f(x)$$

$$u(x, 0) = \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{c} = f(x)$$

$$\cdot \quad \quad \quad f(x) \quad \quad \quad \sum a_n \cos\frac{n\pi x}{c}$$

$$\vdots \quad \quad \quad a_n$$

$$\int_0^c \cos\frac{n\pi x}{c} dx = \frac{c}{n\pi} \sin\frac{n\pi x}{c} = 0 \quad \begin{cases} 0 & n = 1, 2, \dots \\ c & n = 0 \end{cases}$$

$$\int_0^c \cos\frac{n\pi x}{c} \cos\frac{m\pi x}{c} dx = \frac{1}{2} \int_0^c \left(\cos(n+m)\frac{\pi x}{c} + \cos(n-m)\frac{\pi x}{c} \right) dx$$

$$\Rightarrow \quad n, m, m \quad \quad \quad \begin{matrix} n = 1, 2, \dots \\ m = 1, 2, \dots \end{matrix} \quad \begin{cases} 0 & \forall m, n | m \neq n \\ c/2 & n = 0 \end{cases}$$

$$\sum_{n=0}^{\infty} a_n \cos\frac{n\pi x}{c} = f(x) = a_0 + a_1 \cos\frac{n\pi x}{c} + a_2 \cos\frac{2\pi x}{c} + \dots + a_n \cos\frac{n\pi x}{c} = f(x)$$

$$: \quad . \quad c = 0$$

$$a_0 \int_0^c dx + a_1 \int_0^c \cos \frac{nx}{c} dx + \dots + \dots = \int_0^c f(x) dx$$

$$a_0 c = \int_0^c f(x) dx \Rightarrow a_0 = \frac{1}{c} \int_0^c f(x) dx$$

$$. \quad (\quad) \quad \cos \frac{nx}{c} \quad a_1$$

$$a_0 \int_0^c \cos \frac{nx}{c} dx + a_1 \int_0^c \cos \frac{nx}{c} \cos \frac{nx}{c} dx + \dots$$

$$= \int \cos \frac{nx}{c} f(x) dx = a_1 \frac{c}{2} = \int f(x) c \cos \frac{nx}{c} dx$$

:

$$a_1 = \frac{2}{c} \int_0^c f(x) \cos \frac{2nx}{c} dx$$

$$a_n = \frac{2}{c} \int_0^c f(x) \cos \frac{n\pi x}{c} dx \quad \forall n = 1, 2, \dots$$

$$Ku_{xx} = u_t$$

$$u_x(0, t) = 0 \quad u_x(c, t) = 0$$

$$u(x, 0) = ? \quad :$$

$$f(x) = 10x \quad c = 10x \quad . \quad f(x) \quad :$$

.

$$u(x, t) = \sum_{n=0}^{\infty} a_n e^{-\frac{n^2 \pi^2}{c^2} kt} \cos \frac{n\pi x}{c}$$

$$k = \frac{\text{ظرفیت}}{\text{جرم}} = \frac{\text{حرارتی}}{\text{واحد}} \frac{\text{واحد}}{\text{حجم}}$$

$$a_0 = \frac{1}{10} \int_0^c 10x dx \quad a_n = \frac{2}{10} \int_0^c f(x) \cos \frac{n\pi x}{c} dx$$

$$a_0 = 50 \text{ متوسط} \quad a_n = \frac{2}{10} \int_0^{10} 10x \cos \frac{n\pi x}{c} dx$$

$$a_n = \frac{2}{10} \int_0^c x \cos \frac{n\pi}{10} x dx = \frac{2}{10} \left(\frac{u10}{n\pi} \sin \frac{n\pi x}{c} \Big|_0^{10} - \frac{10}{n\pi} \int_0^{10} \sin \frac{n\pi x}{c} dx \right)$$

$$\begin{cases} \text{زوج} & a_n = 0 \\ \text{فرد} & a_n = -\frac{400}{n^2 \pi^2} \end{cases} \quad \text{يا} \quad a_n = \frac{200}{n^2 \pi^2} ((-1)^n - 1)$$

$$a_n = \frac{2}{10} \left(\left(\frac{10}{n\pi} \right)^2 \cos \frac{n\pi x}{10} \Big|_0^{10} \right) \Rightarrow a_n = \frac{200}{n^2 \pi^2} (\cos n\pi - 1)$$

$$u(x, t) = 50 + \sum_{n=1}^{\infty} \frac{200}{n^2 \pi^2} ((-1)^n - 1) e^{-\frac{n^2 \pi^2 k}{10} t} \cos \frac{n\pi x}{10}$$

$$f(x)$$

Separation of Variables

PDE

PDE

$$\begin{aligned} u_{xx}(x, y) &= 0 & u(0, y) &= y^2 & u(1, y) &= 1 \\ 0 < x < 1 & & -\infty < y < \infty \end{aligned}$$

$$u_{xx}(x, y) = 0 \Rightarrow u_x(x, y) = \phi(y) \Rightarrow u(x, y) = \phi(y)x + h_y$$

$$u(0, y) = y^2 \Rightarrow h(y) = y^2 \quad (\phi(y) \times 0)$$

$$u(1, y) = \phi(y) + h(y) = 1 \Rightarrow \phi(y) = 1 - y^2$$

$$u(x, y) = (1 - y^2)x + y^2$$

$$a^2 y_{xx}(x, t) = y_{tt}(x, t)$$

$$\begin{aligned} y(x, 0) &= h(x) & -\infty < x < \infty \\ y_t(x, 0) &= 0 & t > 0 \end{aligned}$$

$$\begin{cases} u = x + at \\ v = x - at \end{cases} \quad \leftarrow :$$

$$(x, t) \rightarrow (v, u)$$

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial t} \quad y(x, t) \rightarrow y(u, v)$$

$$\frac{\partial y}{\partial u} \cdot a + \frac{\partial y}{\partial v} \cdot (-a) = a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial y}{\partial t} \right) \Rightarrow \frac{\partial^2 y}{\partial t^2} = a \frac{\partial}{\partial u} \left(a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v} \right) - a \frac{\partial}{\partial v} \left(a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v} \right)$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial u^2} - a^2 \frac{\partial^2 y}{\partial u \partial v} - a^2 \frac{\partial^2 y}{\partial u \partial v} + a^2 \frac{\partial^2 y}{\partial v^2}$$

$$\Rightarrow a^2 \frac{\partial^2 y}{\partial u^2} - 2a^2 \frac{\partial^2 y}{\partial u \partial v} + a^2 \frac{\partial^2 y}{\partial v^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} \right)$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{\partial y}{\partial u} + \frac{\partial y}{\partial v}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} + \frac{\partial y}{\partial v} \right) + \frac{\partial}{\partial u} \left(\frac{\partial y}{\partial u} + \frac{\partial y}{\partial v} \right) = \frac{\partial^2 y}{\partial u^2} + 2 \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2}$$

PDE

$$a^2 \left(\frac{\partial^2 y}{\partial u^2} + \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2} \right) = a^2 \left(\frac{\partial^2 y}{\partial u^2} - 2 \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2} \right)$$

$$4 \frac{\partial^2 y}{\partial u \partial v} = 0 \Rightarrow \frac{\partial^2 y}{\partial u \partial v} = 0 \Rightarrow \begin{cases} y_u(u, v) = h(u) \\ y_v(u, v) = q(v) \end{cases}$$

:

$$g(u, v) = f(u) + g(v) \quad f'(u) \rightarrow 0$$

$$y(u, v) = f(u) + g(v)$$

$$x + at = u$$

$$x - at = v$$

:

$$y(x, t) = f(x, at) + g(x - at)$$

$$af'(x) - ag(x) = 0 \Rightarrow f'(x) = g'(x)$$

$$f(x) = g(x) + k \quad c$$

$$2f(x) = h(x) + c \Rightarrow f(x) = \frac{1}{2}(h(x) + c)$$

$$2g(x) = h(x) - c \Rightarrow g(x) = \frac{1}{2}(h(x) - c)$$

:

$$\text{پس } y(x, t) = f(x, at) + g(x - at) = \frac{1}{2}(h(a + at) + h(x - at)) \quad \left(\begin{array}{l} \\ \end{array} \right)$$

:

$$y_{tt}(x, t) = a^2 y_{xx}(x, t)$$

$$y(0, t) = y(c, t) = 0 \quad 1$$

$$y(x, 0) = f(x) \quad y_t(x, 0) = 0 \quad 3$$

$$a, c \quad .$$

.

:

$$y(x, t) = X(x)T(t)$$

$$X(x)T''(t) = a^2 X''(x)T(t) \Rightarrow \frac{X''(x)}{X(x)} = \frac{1}{a^2} \frac{T''(t)}{T(t)} = \begin{cases} \lambda^2 \\ 0 \\ -\lambda^2 \end{cases}$$

$$\text{الف} \quad \begin{aligned} \frac{X''(x)}{X(x)} &= \lambda^2 \Rightarrow X''(x) = \lambda^2 X(x) \\ \frac{T''(t)}{T(t)} &= \lambda^2 \rightarrow T''(t) = a^2 \lambda^2 T(t) \end{aligned}$$

$$\begin{cases} X(t) = Ae^{\lambda x} + Be^{-\lambda x} \\ T(t) = Ce^{a\lambda t} + De^{-a\lambda t} \end{cases}$$

$$1 \Rightarrow X(0) = 0 \Rightarrow A + B = 0 \quad \Rightarrow A = 0, B = 0$$

$$\begin{aligned} X(c) = 0 &\Rightarrow Ae^{\lambda c} + Be^{-\lambda c} = 0 \\ \Rightarrow \lambda = 0 &\Rightarrow \begin{cases} X''(x) = 0 \\ T''(t) = 0 \end{cases} \Rightarrow \begin{cases} X(x) = Ax + B \\ T(t) = Ct + D \end{cases} \end{aligned}$$

$$\text{شرایط مرزی} \quad \begin{cases} X(0) = 0 \Rightarrow B = 0 \\ X(c) = 0 \Rightarrow A = 0 \end{cases}$$

$$\text{ج) } \lambda < 0 \quad \begin{cases} X''(x) = -\lambda^2 X(x) \Rightarrow X(x) = B \sin \lambda x + A \cos \lambda x \\ T''(t) = -a^2 \lambda^2 T(t) \Rightarrow T(t) = C \sin a\lambda t + D \cos a\lambda t \end{cases}$$

$$1 \rightarrow X(0) = 0 \Rightarrow A = 0 \quad X_n(x) = \sin \frac{n\lambda}{c} x$$

$$X(c) = 0 \Rightarrow B \sin \lambda c = 0 \Rightarrow \lambda c = n\pi \Rightarrow \lambda = \frac{n\pi}{c} \quad \forall n = 0, 1, \dots$$

:

$$\begin{aligned} T_n(t) &= C \sin \frac{an\pi}{c} + D \cos \frac{an\pi}{c} + * \\ 3: \quad X(x)T'(0) &= 0 \end{aligned}$$

$$* = 0 \Rightarrow T'_n(0) = 0 \Rightarrow C = 0$$

$$T_n(t) = \cos \frac{an\pi}{c} t$$

$$\text{بنابراین} \quad : \quad X_n(x) = \sin \frac{n\pi x}{c} \quad , \quad T_n(t) = \cos \frac{a\pi x}{c} t$$

$$\text{پس} \quad Y_n(x, t) = \sin \frac{n\pi x}{c} \cos \frac{a\pi x}{c} t$$

$$\sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{c} \quad \text{پس} \quad y(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \cos \frac{a\pi x}{c} t$$

b_n

$y(x, t)$

$$\left(\quad t \quad \right)$$

$f(x)$

$$g(x, 0) = f(x)$$

$$y(x, 0) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} = f(x)$$

$$\int_0^c \sin \frac{n\pi x}{c} dx = -\frac{c}{n\pi} \cos \frac{n\pi x}{c} \Big|_0^c = -\frac{c}{n\pi} (\cos n\pi - 1) = \frac{c}{n\pi} ((-1)^{n+1} + 1) = \frac{c}{n\pi} (1 - (-1)^n)$$

$$\int_0^c \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \begin{cases} 0 & n \neq m \\ c/2 & n = m \end{cases}$$

$$\therefore \sin \frac{n\pi x}{c} \sum b_n \sin \frac{n\pi x}{c} = f(x)$$

$$\sum_{n=1}^{\infty} \int_0^c b_n \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

$$c/2 b_m = \int_0^c f(x) \sin \frac{m\pi x}{c} dx \Rightarrow b_m = 2/c \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

$$\begin{cases} f(x) = 0.2x & 0 < x < 10 \\ f(x) = -0.2x + 4 & 10 < x < 20 \end{cases} \quad : \quad f(x)$$

$$= 2/20 \left(2/10 \left(-x \frac{20}{n\pi} \cos \frac{n\pi x}{20} \right) \Big|_0^{10} + \frac{20}{n\pi} \int_0^{10} \cos \frac{n\pi x}{20} dx \right)$$

$$b_n = 2/20 \int_0^{20} f(x) \sin \frac{n\pi x}{20} dx = 2/20 \left[\int_0^{10} 2/10 x \sin \frac{n\pi x}{20} dx + \int_{10}^{20} (-2/20 + 4) \sin \frac{n\pi x}{20} dx \right]$$

$$+ \left(\left(-2/20 x + 4 \right) \left(\frac{-20}{n\pi} \cos \frac{n\pi x}{20} \right) \Big|_{10}^{20} + \frac{20}{n\pi} \int_{10}^{20} \left(\frac{-2}{10} \cos \frac{n\pi x}{20} \right) dx \right)$$

$$\dots + \int_{10}^{20} \frac{-2}{20} x \sin \frac{n\pi x}{20} dx + \int_{10}^{20} 4 \sin \frac{n\pi x}{20} dx$$

$$= 2/20 \int_{10}^{20} x \sin \frac{n\pi x}{20} dx + \frac{4 \times 20}{n\pi} \left(-\cos \frac{n\pi x}{20} \right) \Big|_{10}^{20} = \dots$$

:

:

[a,b] f(x)

$$a < x_1 < x_2 \dots \dots \dots < x_n < b$$

$$x_1, x_2, \dots \dots \dots x_n$$

$$f(x_1^-)f(x_1^+), f(x_2^-), f(x_2^+), \dots \dots \dots, f(x_n^-), f(x_n^+)$$

$$\vdots$$

$$f(x)=\begin{cases} -1 & -1Mx<-0.5 \\ a & -0.5<x<0.5 \\ 1 & 0.5\leq x<1 \end{cases}$$

$$(c,d)\subset (a,b) \qquad (c,d)$$

$$\vdots$$

$$\int\limits_c^d f(x)dx = \int\limits_c^{x_M} f(x)dx + \int\limits_{x_M}^{x_{M+1}} f(x)dx + \dots \dots \dots + \int\limits_{x_{r-1}}^d f(x)dx$$

$$f(x)$$

$$[a,b] \qquad)$$

$$f(x)=a_o/2+\sum_{n=1}^{\infty}a_nCos\frac{n\pi x}{c}$$

$$C \quad 0 \qquad f(x)$$

$$0 < x < \eta \qquad f(x) \qquad :$$

$$f(x)\approx a_o/2+\sum_{n=1}^{\infty}a_nCos\frac{n\pi x}{\pi}$$

$$a_o/2+\sum_{n=1}^{\infty}a_nCosnx=f(x)\Rightarrow a_o/2\int\limits_0^{\eta}dx+\sum_{n=1}^{\infty}a_n\int\limits_0^{\eta}Cos\,nx\,dx=\int\limits_0^{\eta}f(x)dx$$

$$: \qquad a_n$$

$$= a_o / 2 \times \pi + o = \int_o^\eta f(x) dx \Rightarrow a_o = 2 / \pi \int_o^\eta f(x) dx$$

$$a_o / 2 \int_o^\eta \cos mx dx + \sum_{n=1}^{\infty} a_n \int_o^\eta \cos nx \cos mx dx = \int_o^\eta f(x) \cos mx dx$$

$$\sum_{n=1}^{\infty} a_n / 2 \left[\int_o^\eta (\cos(n-m)x - \cos(nn+m)x) dx \right] = \int_o^\eta f(x) \cos mx dx$$

$$\sum_{n=1}^{\infty} a_n / 2 \left[\int_o^\eta (\cos(n-m)x - \cos(nn+m)x) dx \right] = \int_o^\eta f(x) \cos mx dx$$

$$a_n = 2 / \pi \int_o^\eta f(x) \cos nx dx$$

⋮

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad [0, c] \quad f(x)$$

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad n$$

$$\int_o^c \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \int_o^c f(x) \sin \frac{m\pi x}{c} dx \quad b_n$$

$$b_n = \frac{2}{c} \int_o^c f(x) \sin \frac{n\pi x}{c} dx \Rightarrow \int_o^c f(x) \sin \frac{m\pi x}{c} dx = \int_o^c f(x) \sin \frac{m\pi x}{c} dx$$

$$(0, \pi) \quad f(x) = x$$

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \Rightarrow f(x) = \sum_{n=1}^{\infty} b_n \sin \pi x$$

$$b_n = 2 / \pi \int_o^\pi x \sin nx dx = \frac{-x}{n} \cos nx \Big|_o^\pi + \frac{1}{n} \int_o^\pi \cos nx dx$$

$$= 2 / \pi \left(\pi / n (-1)^{n+1} - o + \frac{1}{n^2} \sin x \Big|_o^\pi \right) = \left(\frac{(-1)^{n+1} \times \pi}{n} \right) \times 2 / \pi = \frac{2(-1)^{n+1}}{n}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \times 2}{n} \sin nx = 2 \sin x - \sin 2x + \frac{2}{3} \sin 3x \dots$$

$$f(x) = x \quad [0, \pi] \quad \sum_{n=1}^{\infty} b_n \sin nx \quad f(x) \quad \sin nx$$

$$f(x) = a_0/2 + \sum a_n \cos nx \quad (\quad) \quad \pi/2 \quad - \quad :$$

$$(-c, c) \quad f(x) \quad :$$

$$f(x) = h(x) + g(x) \quad \text{and} \quad g(-x) = -g(x) \quad , \quad h(-x) = h(x)$$

$$h(x) = \frac{f(x) + f(-x)}{2} \quad g(x) = \frac{f(x) - f(-x)}{2} \quad h(x)$$

$$(-c, c) \quad h(x)$$

$$h(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} \quad a_n = 2/c \int_0^c h(x) \cos \frac{n\pi x}{c} dx$$

$$f(x) = h(x) + g(x) \quad (-c, c)$$

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad \text{and} \quad x \in (-c, c) \quad (-c, c) \quad f(x)$$

$$a_n = 2/c \int_0^c \frac{f(x) + f(-x)}{2} \cos \frac{n\pi x}{c} dx = 1/c \left[\int_0^c f(x) \cos \frac{n\pi x}{c} dx + \int_0^c f(-x) \cos \frac{n\pi x}{c} dx \right]$$

$$= 1/c \left[\int_0^c f(x) \cos \frac{n\pi x}{c} dx + \int_{-c}^0 f(x) \cos \frac{n\pi x}{c} dx \right] = 1/c \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{c} + b_n \sin \frac{n\pi x}{c} \right)$$

$$a_n = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx \quad b_n = \frac{1}{c} \int_{-c}^c f(x) \sin \frac{n\pi x}{c} dx$$

$$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 1 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases} \quad x \leftarrow (-\pi, \pi) :$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^N (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin \frac{n\pi x}{\pi} dx$$

$$\frac{1}{\pi} \int_{-\pi/2}^{\pi} 1 \times \cos nx dx = \frac{1}{n\pi} \sin nx \Big|_{-\pi/2}^{\pi/2} = \frac{1}{n\pi} \left(\sin \frac{n\pi}{2} + \sin \frac{n\pi}{2} \right) = \frac{1}{n\pi} \times 2 \sin \frac{n\pi}{2}$$

$$\text{Thus } b_n : \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sin nx dx = 0$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \cos nx \quad \int_{-\pi/2}^{\pi/2} 1 dx = 1 \quad a_0 =$$

$$f(x) \quad (-\pi, \pi) \quad f(x) :$$

$$f(x)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx \quad f(x_0)$$

$$f(x) = x \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$a_n = 0 \quad f(x)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx = \frac{2}{\pi} \left(-\frac{x}{n} \cos nx + \frac{1}{n^2} \sin nx \right) \Big|_0^{\pi}$$

$$= \frac{2}{\pi} \left(-\frac{\pi}{n} \cos n\pi + \frac{1}{n^2} \sin n\pi \right) = \frac{2(-1)^{n+1}}{n}$$

$$f(x) \cong \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$= 2 \left(\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \dots \right)$$

$$f(x) = f(x) \quad f(-\pi) = 0 \quad f(\pi) = 0 \quad -\pi, \pi, f$$

$$\begin{aligned} \frac{2}{\pi} \int_0^{\pi} S_N(x) \cos nx &= a_n \\ S_N(x) &= \frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \end{aligned}$$

$$\begin{aligned} \frac{2}{\pi} \int_0^{\pi} (S_N(x))^2 dx \\ \frac{2}{\pi} \int_0^{\pi} \left(\frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \right) S_N(x) dx &= \frac{2}{\pi} \int_0^{\pi} \frac{a_0}{2} S_N(x) dx + \frac{2}{\pi} \sum_{n=1}^N a_n \int_0^{\pi} S_N(x) \cos nx dx \\ &= \frac{2}{\pi} \frac{a_0^2}{4} \int_0^{\pi} dx + \frac{2}{\pi} \sum_{n=1}^N a_n^2 \times \frac{\pi}{2} = \left(\frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \right) \end{aligned}$$

$$S_N(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$\frac{2}{\pi} \int_0^{\pi} S_N(x) \sin mx dx = b_m$$

$$\begin{aligned} \frac{2}{\pi} \int_0^{\pi} [S_N(x)]^2 dx &= \sum_{n=1}^N \frac{2}{\pi} \int_0^{\pi} S_N(x) b_n \sin nx dx \\ &= \sum_{n=1}^N b_n \times \frac{2}{\pi} \int_0^{\pi} S_N(x) \sin nx dx = \sum_{n=1}^N (b_n)^2 \end{aligned}$$

$$E = \frac{2}{\pi} \int_0^{\pi} (f(x) - S_N(x))^2 dx \geq 0$$

$$\frac{2}{\pi} \int_0^{\pi} f^2(x) dx - \frac{4}{\pi} \int_0^{\pi} f(x) S_N(x) dx + \frac{2}{\pi} \int_0^{\pi} (S_N(x))^2 dx$$

$$(0, \pi) \quad f(x)$$

$$\eta \leq 0$$

$$\int_0^{\pi} f^2(x) dx$$

$$\frac{2}{\pi} \int_0^{\pi} f(x) S_N(x) dx \quad \left(\quad \right)$$

:

$$\begin{aligned} \int_0^\pi f(x) \left(\frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \right) dx &= \frac{1}{2} \int_0^\pi f(x) dx + \sum_{n=1}^N a_n \int_0^\pi f(x) \cos nx dx \\ &= \frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \end{aligned}$$

:

$$\int_0^\pi f(x) \left(\sum_{n=1}^N b_n \sin nx \right) dx = \sum_{n=1}^N b_n \int_0^\pi f(x) \sin nx dx = \sum_{n=1}^N b_n^2$$

$$\int_0^\pi f^2(x) dx = \frac{a_0^2}{2} + \sum_{n=1}^\infty (a_n^2 + b_n^2)$$

$$\int_0^\pi f^2(x) dx \geq \left(\frac{a_0}{2} + \sum_{n=1}^N a_n^2 \right)$$

$$\sum_{n=1}^N b_n^2 \leq \int_0^\pi f^2(x) dx$$

$$\left(\sum_{n=1}^N \right) \leq N$$

:

$$\frac{a_0}{2} + \sum_{n=1}^N a_n^2 \geq \int_0^\pi f^2(x) dx$$

:

$$\frac{a_0^2}{2} + \sum_{n=1}^\infty a_n^2$$

$$\lim_{n \rightarrow \infty} a_n = 0$$

:

$$\int_0^\pi f^2(x) dx \geq 2 \times \left(\sum_{n=1}^N b_n^2 + \sum_{n=1}^N a_n^2 \right) \geq 0$$

$$\sum_{n=1}^N b_n^2 \leq \int_0^\pi f^2(x) dx$$

$$\lim_{n \rightarrow \infty} b_n = 0$$

.

$$a_n, b_n$$

:

$$(-n, n)$$

$$f(x)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^\infty (a_n \cos nx + b_n \sin nx)$$

:

$$b_n = \frac{1}{n} \int_{-n}^n f(x) \sin nx dx$$

$$f(x)$$

$$f(x)$$

$$f(x)$$

$$\lim ax = 0 \quad , \quad \lim bx = 0$$

$$a_0^2/2 + \sum_{n=1}^N a_n^2 \leq 1/R \int_{-R}^R f(x) \cos^2 dx \geq 1.$$

$$\sum_{n=1}^N b_n^2 \leq \frac{1}{R} \int_{-R}^R f^2 \cos^2(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \beta(x) \cos nx \, dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \beta(x) \sin nx \, dx$$

$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f'(x) \cos nx dx, \quad \beta_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f'(x) \sin nx dx$$

$$S_N \sum_{n=1}^{\infty} \sqrt{a_n 2 + b_n 2}$$

$$S_N = \left(\sum_{n=1}^N \sqrt{a_n 2 + b_n 2} \right)^2$$

$$S_N = \left(\sum_{n=1}^N \left(a_n^2 + b_n^2 \right)^{\frac{1}{2}} \right)^2 = \left(\sum_{n=1}^N \left(\frac{\alpha_n^2}{n^2} + \frac{\beta_n^2}{n^2} \right)^{\frac{1}{2}} \right)^2 =$$

$$S_N = \left(\sum_{n=1}^N \left(\frac{1}{n^2} \left(a_n^2 + b_n^2 \right)^{\frac{1}{2}} \right) \right)^2 < \left(\sum_{n=1}^N \frac{1}{n^2} \right) \left(\sum_{n=1}^N \left(\alpha_n^2 + \beta_n^2 \right) \right)$$

$$\sum_{n=1}^N (\alpha_n^2 + \beta_n^2) \qquad \sum_{n=1}^N 1/n^2$$

•

•

$$\frac{1}{\eta} \int_{-\eta}^{\eta} f^2(x) dx = \frac{a_o^2}{2} + \sum_{n=1}^{\infty} (an^2 + bn^2) \quad ($$

$$f'(x) = \sum (-n a_n \sin nx + n b_n \cos nx)$$

$$\begin{cases} x(0) = 0 & x(a) = 0 \\ y(0) = 0 & y(b) = 0 \end{cases} :$$

$$\frac{X''}{X} = -\eta^2 \Rightarrow X''(x) = -\eta^2 X \Rightarrow X = B \sin \eta x + A \cos \eta x$$

$$X(a) = 0 \rightarrow A = 0$$

$$X(a) = 0 \rightarrow 0 = B \sin \eta a \Rightarrow \eta a = x \eta \Rightarrow \eta = \frac{n \eta}{a}$$

$$\Rightarrow X_n(x) = \sin \left(\frac{n \eta}{a} x \right)$$

$$\frac{y''}{y} - \lambda^2 = -\eta^2 \Rightarrow \frac{y''}{y} = \lambda^2 - \eta^2 \Rightarrow y'' = (\lambda^2 - \eta^2)y$$

$$\lambda^2 - \eta^2$$

$$y(0) = 0$$

$$y(b) = 0$$

$$y'' = -(\eta^2 - \lambda^2)y$$

$$y = C \sqrt{\eta^2 - \lambda^2} y + D \sin \sqrt{\eta^2 - \lambda^2} y$$

$$\sqrt{\eta^2 - \lambda^2} = \frac{m \eta}{b} \Rightarrow \eta^2 - \lambda^2 = \left(\frac{m \eta}{b} \right)^2 \Rightarrow \lambda^2 = \left(\frac{n \eta}{a} \right)^2 - \left(\frac{m \eta}{b} \right)^2$$

$$\Rightarrow \lambda = \eta \sqrt{\frac{n^2}{a^2} - \frac{m^2}{b^2}}$$

$$\gamma_m = \sin \left(\frac{m \eta}{b} y \right) \quad \eta \lambda$$

$$\begin{aligned} &: \quad Z_3 = 3 - 2\lambda \quad Z_2 = 1 - \lambda \quad z_1 = 2\lambda \quad - \\ &3i \quad \left(1 + \sqrt{3}i \right) \quad \left(-1 + i \right) \quad - \\ &: \quad - \end{aligned}$$

$$3i \quad \left(\quad \quad \quad 1+\sqrt{3}i \quad \left(\quad \quad \quad -1+i \quad \left(\right.$$

$$: \quad \quad \quad Z_3 = se^{\frac{i}{u}} \quad , \quad Z_2 = \sqrt{2}e^{\frac{\pi}{3}} \quad , \quad Z_1 = 2e^{i\pi} \quad -$$

$$Z^4-2Z^2+4=0 \quad \left(\quad \quad \quad Z_1\overline{Z_2} \quad \left(\quad \quad \quad \frac{Z_1}{Z_3} \quad \left(\right. \right. \\ \left. \left. \cos 3\theta \quad , \quad \sin 3\theta \right. \right. \quad -$$

$$: \quad \quad \quad -$$

$$\operatorname{Im}(iz)=\operatorname{Re} z \quad \left(\quad \quad \quad \operatorname{Re}(iz)=-\operatorname{Im} z \quad \left(\right.$$

$$\overline{\overline{Z}+3i}=z-3i \quad \left(\quad \quad \quad \overline{iz}=-i\overline{z} \quad \left(\right.$$

$$: \quad \quad \quad -$$

$$) \operatorname{Re}(\overline{z}-i)=z$$

$$) \left| z-i\right| =\left| z+i\right|$$

$$\quad \quad \quad . \quad \quad \quad Z_1Z_2=0 \quad -$$

$$: \quad \quad \quad -$$

$$\left(\quad \quad \quad Z^{\frac{2}{3}}+3i=0 \quad \left(\quad \quad \quad Z^2+4=0 \quad \left(\right.$$

$$\quad \quad \quad z^2+z^{-2}=2 \quad \quad \quad x^2-y^2=1 \quad -$$

$$-$$

$$(1+z)^n=1+\frac{n}{i}z+\frac{n(n-1)}{2i}z^2+.....\frac{n(n-1)(n-2)+....(n-k+1)}{ki}z^x+.....+z^n$$

$$. \quad \quad \quad n$$

$$z_1-z_2 \quad , \quad z_1+z_2$$

$$.$$

$$) z_{1=} = (-3,1) \quad z_2 = (1,4)$$

$$) z_1 = x_1 + iy_1 \quad z_{2=} = x_1 - iy_1$$

$$: \quad \quad \quad -$$

$$\left| (2\overline{z}+5)(\sqrt{2}-i) \right| = \sqrt{3} |2z+5|$$

$$\left| \operatorname{Re} z \right| + \left| \operatorname{Im} z \right| < \sqrt{2} \left| z \right| \quad : \quad -$$

$$: \quad \quad \quad \operatorname{arg} z \quad -$$

$$Z=\frac{-2}{1+\sqrt{3}i}$$

$$Z=\frac{i}{-2-2i}$$

$$(\hspace{10cm})($$

$$\mathbf{g},\mathbf{f}:$$

$$\vec{f}=(a_1,a_2,a_3)\qquad\qquad\vec{g}:(b_1,b_2,b_3)$$

$$:$$

$$\vec{f}.\vec{g}=a_1b_1+a_2b_2+a_3b_3\qquad\qquad\left|f\right|^2=f.f=a_1^2+a_2^2+a_3^2$$

$$\vec{f}.\vec{g}=\left|\vec{f}\right|\left|\vec{g}\right|.Cos\theta\qquad\qquad\left|\vec{f}-\vec{g}\right|=\left(\left(a_1-b_1\right)^2+\left(a_2-b_2\right)^2+\left(a_3-b_3\right)\right)^{\frac{1}{2}}$$

$$f+g\Rightarrow\vec{f}.\vec{g}=0\qquad\qquad\left|f\right|=1\Rightarrow f=\text{بردار نرمال}$$

$$:\hspace{10cm}\phi_n,\ldots,\phi_2,\phi_1$$

$$<\phi_i,\phi_j>=0\quad,\quad<\phi_i\phi_i>=\left|\phi_i\right|^2$$

$$\frac{\phi_n}{\left|\phi_n\right|},\ldots,\frac{\phi_2}{\left|\phi_2\right|},\frac{\phi_1}{\left|\phi_1\right|}$$

$$\hspace{10cm}\phi_n,\ldots\phi_1$$

$$f=c_1\phi_1+c_2\phi_2,\ldots,c_n\phi_n\hspace{10cm}.$$

$$:$$

$$C_K<\phi_k\phi_k>=<f.\phi_k>\Rightarrow C_k=\frac{1}{\left\|\phi_k\right\|}<f,\phi_k>$$

$$:$$

$$\Delta x\hspace{10cm}(\text{pwc})\hspace{10cm}-\hspace{10cm}(a,b)\hspace{10cm}f,\,g$$

$$:$$

$$f=(f_1,f_2,\ldots,f_n)$$

$$g=(g_1,g_2,\ldots,g_n)$$

:f, g

$$(f, g) = \sum f_k g_k \Delta x_k$$

$$\lim_{\Delta x \rightarrow 0} (f, g) = \int_a^b f(x)g(x)dx$$

Fundamental interval

a, b

:

$$(f, g) = (g, f)$$

$$(f, g + h) = (f, g) + (f, h)$$

$$(cf, g) = c(f, g)$$

$$(f, f) = \|f\|^2 \geq 0 \rightarrow \|f\| = \left(\int_a^b f^2(x) dx \right)^{1/2}$$

$$(f, g) = 0 \Rightarrow f \perp g$$

$$\|f - g\|^2 = \int_a^b (f - g)^2 dx$$

$$(f, g) = 1 \Rightarrow f \text{ and } g \text{ are orthonormal}$$

:

$$\langle \Psi_i(x), \Psi_j(x) \rangle = \frac{2}{c} \langle \Psi_i \Psi_j \rangle = \delta(i, j) \quad 0 < x < c \quad \Psi_n(x) = \sin \frac{n\pi x}{c}$$

:

$$\phi_i = \sqrt{\frac{2}{c}} \sin \frac{n\pi x}{c}$$

$$\phi_0(x) = \frac{1}{\sqrt{c}} \quad , \quad \phi_n(x) = \sqrt{\frac{2}{c}} \cos \frac{n\pi x}{c}$$

:

$$0 < x < c$$

$$\phi_0(x) = \frac{1}{\sqrt{c}} \sin \frac{n\pi x}{c} \quad , \quad \phi_{2n-1} = \frac{1}{\sqrt{c}} \cos \frac{n\pi x}{c} \quad , \quad \phi_0 = \frac{1}{\sqrt{2c}} \quad (-C, C)$$

:

$$f = c_0 \phi_0 + c_1 \phi_1 + \dots + c_k \phi_k + \dots + c_n \phi_n$$

$$C_k = \langle f, \phi_k \rangle \quad k = 0, 1, \dots, \quad C_k = \int_a^b f(x) \phi_k(x) dx$$

:

$$f(x)=C_0\frac{1}{\sqrt{2c}}+C_1\frac{1}{\sqrt{2}}Cos\frac{n\pi x}{c}+C_2\frac{1}{\sqrt{2}}Sin\frac{n\pi x}{c}+.....$$

$$C_k=<f(x).\frac{1}{\sqrt{c}}Cos\frac{n\pi x}{c}>=\frac{1}{\sqrt{c}}\int\limits_{-c}^cf(x)Cos\frac{n\pi x}{c}dx$$

$$a_k=\frac{ck}{\sqrt{c}}=1/c\int\limits_{-c}^cf(x)Cos\frac{n\pi x}{c}dx\;.$$

:

$$\phi_n(x),.....,\phi_2(x),\phi_1(x)$$

$$<\phi_m(x)\phi_k(x)>=\begin{cases}0&k\neq m\\c_k^2=1&k=m\end{cases}\quad k,m=1,2,.....$$

$$\phi_n(x),.....\phi_1(x)$$

$$\phi(x)$$

$$\phi(x)=\delta_1\phi_1(x)+.....+\delta_n\phi_n(x)$$

$$f(x)$$

$$\phi_{(x)}$$

$$k=1,2,.....n$$

$$\phi_k$$

$$f(x)$$

.

$$f\;f(x)$$

$$\phi(x)$$

$$\delta_n,.....\delta_2,\delta_1$$

:

$$|f(x)-\phi(x)|$$

$$|f(x)-\phi(x)|$$

$$E=|f(x)-\phi(x)|=\left(\int\limits_a^bf(x)-\phi(x)^2dx\right)^{1/2}$$

$$E^2=|f(x)-\phi(x)|^2=\int(f(x))-\phi(x)^2\;dx$$

$$\int\limits_a^bf^2(x)dx-2\int\limits_a^bf(x)dx+\int\limits_a^b\phi^2(x)dx$$

$$=|f(x)|^2-2\int\limits_a^bf(x)\sum_{k=1}^n\delta_k\phi_k(x)dx+\int\limits_a^b\left(\sum_{k=1}^n\delta_k\phi_k(x)\right)^2dx$$

$$=|f(x)|^2-2\sum_{k=1}^n\delta_k\int\limits_a^bf(x)\phi_k(x)dx+\int\limits_a^b\sum_{k=1}^n\sum_{k=1}^n\delta_k\phi_k(x)\delta_\ell\phi_\ell(x)dx$$

.

$$\delta_k$$

.

$$\text{Min}$$

$$E^2$$

$$\text{Min}$$

$$\delta_k$$

$$\delta_x$$

$$\phi(x)$$

$$|f|^2E^2\sum[\delta_k]^2+2\sum_{k=1}^n(\delta_k=|f_k|^2)+\sum_{k\neq l=1}^n(\delta_{kk}^2-2c_k\delta_k+c_k^2)-\sum_{k=1}^nc_k^2$$

$$\delta_k = ck$$

$$= \left| f(x) \right|^2 - 2 \sum_{k=1}^n \delta_K c_k + \sum_{k=1}^n \delta_k^2 (1)$$

$$c_k = \langle f(x) \phi_k(x) \rangle$$

$$\delta_k = c_k \Rightarrow E^2 = \left\| f \right|^2 - \sum_{k=1}^n C_k^2$$

$$: \quad \left(\right) \quad E^2 > 0$$

$$\left| f(x) \right|^2 \geq \sum_{k=1}^n C_k^2 \qquad \forall_n \in N \qquad (n= \infty \qquad)$$

$$n \rightarrow \infty \qquad \sum_{k=1}^n ck^2 \qquad \lim C_n=0 \qquad n \rightarrow \infty$$

$$\left| f(x) \right|^2 = \sum_{k=1}^n C_k^2 \qquad : \qquad . \qquad \left| f(x) \right|^2$$

$$: \qquad (a,b) \qquad f \qquad (a,b) \qquad \{\phi_k(x)\}$$

$$\sum_{k=1}^n C_k^2 = \left| f \right|^2$$

$$0=$$

$$\phi_0 = \frac{1}{\sqrt{2\pi}}, \quad \phi_{2n-1}(x) = \frac{1}{\sqrt{\pi}} \cos nx, \quad \phi_{2n}(x) = \frac{1}{\sqrt{\pi}} \sin nx$$

$$(-\pi,\pi)$$

$$b_n \qquad : \qquad C$$

$$a_n \qquad : \qquad C$$

$$:$$

$$p(x) \qquad (a, b) \qquad \phi_k(x)$$

$$\int\limits_a^b P(x)\phi_k(x)\phi_m(x)dx = \begin{cases} 0 & m \neq k \\ cte & m = k \end{cases}$$

$$\sqrt{p(x)}\phi_k(x)$$

$$\phi_{mn}(x,y)$$

$$\int\int_R\phi_{mn}(x,y)-\phi_{ek}(x,y)dxdy=\begin{cases}0&m\neq e,n\neq k\\c_{mn}^2&m=e,n=k\end{cases}$$

$$\phi_{mn}(x,y)$$

$$\omega = u + i v$$

$$\int\limits_a^b\omega(t)dt=\int\limits_a^bu(t)dt+i\int\limits_a^bv(t)dx$$

$$\overline{\omega}(t)=u(t)-iv(t)$$

$$\begin{array}{c} \vdots \\ \langle \omega_m(t), \omega_k(t) \rangle = \int_a^b \omega_m(t) \overline{\omega_k(t)} \quad dt = \begin{cases} 0 & m \neq k \\ \int (u_m^2(t) + v_m^2(t)) dt \end{cases} \end{array}$$

$$(-2,2)$$

$$\vdots$$

$$f(x)=a_0/2+\sum_{n=1}^{\infty}a_nCos\frac{n\eta x}{c}+b_nSin\frac{n\eta x}{c}$$

$$a_0=1/c\int\limits_{-c}^cf(x)dx\qquad a_n=1/c\int\limits_{-c}^cf(x)Cos\frac{n\eta x}{c}dx$$

$$b_n=1/c\int\limits_{-c}^cf(x)Sin\frac{n\eta x}{c}dx$$

$$\vdots$$

$$\frac{1}{2c}\int\limits_{-c}^cf(s)ds+\sum_{n=1}^{\infty}\frac{1}{c}\bigg[\int\limits_{-c}^cf(s)\cos\frac{n\pi s}{c}\cos\frac{n\pi x}{c}ds+\int\limits_{-c}^cf(s)\sin\frac{n\pi s}{c}\sin\frac{n\pi x}{c}ds\bigg]$$

$$= \frac{1}{2c} \int_{-c}^c f(s) ds + \frac{1}{c} \sum_{n=1}^{\infty} \int_{-c}^c f(s) \cos \frac{n\eta}{c} (s-x) dx$$

:	$(-\infty, \infty)$	f(x)
:		

$$\int_{-\infty}^{\infty} |f(s)| ds$$

$$\Delta\alpha = \eta/c \quad :$$

$$\frac{1}{c} \sum_{n=1}^n \int_{-\infty}^{+\infty} f(s) \cos(n\Delta\alpha(x-s)) ds = \frac{1}{\eta} \sum_{n=1}^{\infty} \Delta\alpha \int_{-\infty}^{+\infty} f(s) \cos(n\Delta\alpha(x-s)) ds$$

$$\frac{1}{\eta} \sum_{n=1}^{\infty} f(n\Delta\alpha) \Delta x$$

$$1 = \frac{1}{\eta} \int_0^{\infty} F(\alpha, x) d\alpha = \frac{1}{\eta} \int_0^{\infty} \int_{-\infty}^{\infty} f(s) \cos(\alpha(x-s)) ds d\alpha$$

$$\sum_{n=1}^{\infty} f(n\Delta\alpha)\Delta x = \int_o^x f(x)dx = f(x) = \int_0^{\infty} (A(\alpha)\cos \alpha x + B(\alpha)\sin \alpha x)$$

$$\mathcal{C} \rightarrow \infty$$

$$f(x) = \frac{1}{\eta} \int_0^{\infty} \int_{-\infty}^{\infty} f(s) \cos \alpha(s-x) ds dx \quad -\infty < x < \infty$$

$$A(\alpha) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(u) \cos \alpha u du \quad B(\alpha) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \sin \alpha u du$$

$$f(x) = e^{-2|x|} \quad :$$

$$A(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(s) \cos \alpha s ds \quad B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(s) \sin \alpha s ds$$

$$|bn|n^4 < M \rightarrow \left| 4bn \sin \frac{n\eta x}{c} \right| \rightarrow \left| b_n \sin \frac{n\eta x}{c} \right| \leq |b_n|$$

$$\leq \sum \left| b_n \right| \leq \frac{M}{n^4} = M \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$f(x) = \int (A(\alpha) \cos x + B(\alpha) \sin \alpha x) d\alpha$$

$$A(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(x) \cos \alpha x dx, B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(x) \sin \alpha x dx$$

$$f(x) = e^{-|x|} \quad -\infty < x < \infty \quad B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} e^{-|x|} \sin \alpha x dx$$

$$f(x) = \int_0^{\infty} (A(\alpha) \cos \alpha x + B(\alpha) \sin \alpha x) d\alpha \quad \text{Re} \int_0^{\infty} e^{-x} e^{i\alpha x} \quad B(\alpha) = 0, A(\alpha) = \frac{2}{\eta} \int_0^{\infty} e^{-x} \cos \alpha x dx$$

$$A(\alpha) = \frac{1}{\eta} \int e^{-|x|} \cos \alpha x dx$$

$$= \frac{2}{\eta} \times \frac{1}{\eta} \left[\frac{1}{\alpha} - \frac{1}{\alpha} A(\alpha) \times \frac{\eta}{2} \right]$$

$$\Rightarrow A(\alpha) = \frac{2}{\eta} \left(\frac{1}{\alpha^2} (1 - A(\alpha)) \right) \Rightarrow \frac{\eta \alpha^2 (A(\alpha))}{2} = 1 - A(\alpha) \frac{\eta}{2}$$

$$\Rightarrow A(\alpha) = \frac{2}{\eta(\alpha^2 + 1)}$$

$$e^{-|x|} = \int_0^{\infty} \frac{2}{\eta(\alpha^2 + 1)} \cos \alpha x dx \Rightarrow$$

$$\frac{2}{R} \int \frac{\cos 5x}{1 + x^2} dx = \frac{R}{2} e^{-5}$$

$$\int \frac{\cos 5\alpha}{1 + \alpha^2} d\alpha = \frac{\pi}{2} e^{-5} \rightarrow \alpha = x$$

$$\int_0^{\infty} \frac{\sin 2\pi x}{1 + x^2} dx$$

$$f(x) = \begin{cases} -e^{-x} & x > 0 \\ -e^{-x} & x < 0 \end{cases} \quad \text{جواب} \quad = \frac{\pi}{2} e^{-2\pi}$$

$$\frac{2}{R} \int \frac{\cos 5x}{1 + x^2} dx = \frac{R}{2} e^{-5} \rightarrow \alpha = x$$

$$f(x_0) = \int_0^{\infty} A(\alpha) \cos \alpha x_0 + B(\alpha) \sin \alpha x_0 d\alpha$$

$$u_t(x,t)=ku_{xx}(x,t)$$

$$u(0,t)=0 \qquad u(x,0)=F(x) \qquad x>0 \qquad t>0$$

$$\cdot \qquad \qquad \qquad \mathbf{x}$$

$$\cdot$$

$$u(x,t)X(x)T(t) \rightarrow \frac{T'(t)}{kt(t)} = \frac{X''(x)}{X(x)} = \begin{cases} x^2 \\ 0 \\ -\lambda^2 \end{cases} \Rightarrow \text{عق عع عع}$$

$$U(x_1)=0\rightarrow X(0)=0$$

$$X(x)=A\cos(\lambda x)+B\sin(\lambda x)$$

$$X(x)=\sin \lambda x$$

$$T(t)=e-\lambda zkt$$

$$U(x,t)=\int_0^{\infty}(B\lambda e^{-k\lambda t}+ \sin \lambda x)dx \Rightarrow \int_{-\infty}^{\infty}B(\lambda)\sin \lambda x d\lambda = f(x)$$

$$B(\lambda)=\frac{2}{\pi}\int_{-\infty}^{\infty}f(x)\sin \lambda xdx$$

$$\vdots$$

$$\vdots$$

$$\Delta^2 u = p^2 \frac{du^2}{dp^2} + p \frac{du}{dp} + (\lambda p^2 - v^2) y = 0$$

$$\vdots$$

$$x=\sqrt{\lambda}p$$

$$x/\lambda \times \frac{\partial^2 u}{\partial p^2} \cdot \frac{\partial x^2}{\partial p^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial p^2} = \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial p} \right) = \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial p} \right) \cdot \frac{ex}{ep}$$

$$\vdots$$

$$= \frac{\partial}{\partial x} \left(\sqrt{\lambda} \frac{\partial u}{\partial x} \right) \sqrt{\lambda} = \lambda \frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow \frac{x^2}{\lambda} \cdot \lambda \frac{d^2 y}{dx^2} + \frac{x}{\lambda} \cdot \sqrt{\lambda} \frac{dy}{dx} + (x^2 - v^2) y = 0$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} \sum_{k=0}^{\infty} a_k x^k (x^2 - v^2) y = 0$$

$$x = 0$$

$$x^2 y'' + xy' + (x^2 - v^2)y = 0 \quad y'' + \frac{1}{x}y' + \frac{x^2 - v^2}{x^2}y = 0$$

$$x=0$$

$$x^2 \sum_{k=0}^{\infty} (r+k)(r+k-1)a_k x^{r+k-2} + x \sum_{k=0}^{\infty} (r+k)a_k x^{(r+k-1)}$$

$$t(x^2 + v^2) \sum_{k=0}^{\infty} (r+k)(r+k-1)a_k x^{r+k} = 0$$

$$n=1,2,\dots, \quad v=n$$

$$\sum ((r+k)(r+k-1)avx^{r+k} + (r+k)akx^{r+k} - n^2akx^{r+k}) + \sum akx^{r+k+2} = 0$$

$$\sum_{k=0}^{\infty} x^{r+k} ((r+k)(r+k-1) + (r+k) - n^2ak + akx^2) = 0$$

$$: (r-n)(r+n)a_0 + (r-n+1)(r+n+1)a_1x + \sum ((r-n+k)(r+n+k)a_k + a_k - 2)x^k = 0$$

$$\Rightarrow a_0 \quad (r+n)(r+n)=0$$

$$(r-n+1)(r-n+1)=0 \quad a_1=0$$

$$* \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$* \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$r=n \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$a_1, a_3, a_{2m} + 1 = 0 \qquad \qquad \qquad k$$

$$a_0 \neq 0$$

$$a_2 = \frac{-1}{2(2n+2)} a_0, a_4 = \frac{1}{4(2n+2)(2)(2n+2)}$$

$$a_{2k} = \frac{(-1)^k}{k!(n+1)(n+2)(n+k)2^{2k}}$$

$$a_0$$

$$a_0 = \frac{1}{n!2^n} \qquad \qquad \qquad a_{2k} = \frac{(-1)^k}{k!(n+k)2^{n+2k}} \qquad \qquad \qquad :$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{n+2k} \qquad \qquad \qquad :$$

$$y_n(x)$$

$$y_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{n+2k}$$

$$y_n(x)$$

$$x^2 y'' + xy' + (x^2 - n^2)y = 0 \rightarrow y = y_n(x)$$

$$:$$

$$x^2 y'' + xy' + x^2 y = 0 \rightarrow y = y_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{x}{2}\right)^{2k}$$

$$y_0(x)$$

$$y_0(x) = 1 - \left(\frac{x}{2}\right)^2 + \frac{1}{(2!)^2} \left(\frac{x}{2}\right)^4 - \frac{(1/3!)^2}{(2!)^2} \left(\frac{x}{2}\right)^6 + \dots \dots \dots y_0$$

$$:$$

$$\vdots$$

$$1) \cos(x \sin \theta) = y_0(x) + \sum_{n=1}^{\infty} (1 + (-1)^n) y_n(x) \cos \phi$$

$$2) \sin(x \sin \theta) = \sum_{n=1}^{\infty} (1 - (-1)^n) y_n(x) \sin \phi$$

$$y_{2n}(x) = \frac{1}{R} \int_0^R \sin(x \sin \phi) \sin(2x - 1) \phi d\phi$$

$$) \quad y_n(x) = \frac{1}{R} \int_0^R \cos(n\phi - x \sin \phi) d\phi$$

$$y_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{K!(x+k)} \left(\frac{x}{2}\right)^{n+2k}$$

$$x^{-n} y_n(x) = \frac{1}{2^n} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{2k}$$

$$4) \frac{d}{dx} x^{-n} y_n(x) = -n x^{-n-1} y_n(x) + x^{-n} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \frac{(n+2x)}{2} \left(\frac{x}{2}\right)^{n-2k-1}$$

$$-x^{-n} y_{n+1}(x) \quad \text{بجاء} \quad y'(0) = -y_1(x)$$

$$y_0(x)$$

$$Y_0(x) = \frac{2}{\pi} [(\log \frac{x}{3} - \nu) J_0(x)] + \frac{x^4}{x^2} - \frac{x^4}{x^2 4^2} (1 + \frac{1}{2}) + \frac{x^6}{x^2 4^2 6^2} (1 + \frac{1}{2} + \frac{1}{3}) + \dots$$

$$R(V) = \frac{R(v+1)}{v}$$

$$\frac{V}{V} \left(\frac{\vec{3}}{2} \right)^{-1^t} = \frac{1^t}{\sqrt{n}} \frac{V}{2} \rightarrow -1^t = \min$$

$$\vdots$$

$$1) \cos(x \sin \theta) = y_0(x) + \sum_{n=1}^{\infty} (1 + (-1)^n) y_n(x) \cos \phi$$

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$$-x^{-n}y_{n+1}(x) \qquad \text{بسط} \qquad y'(0) = -y_1(x)$$

:

$$1) xy_n(x) = ny_n(x) - xy_{n+1}(x)$$

$$2) xy_n(x) = -ny_n(x) + xy_{n-1}(x)$$

$$xy_{n+1}(x) = 2ny_n(x) - xy_{n+1}$$

:

$$. \qquad \qquad \qquad (\qquad \qquad) \qquad \qquad \qquad y_n(x)$$

$$y_n(x)=0 \rightarrow \qquad \qquad \qquad = \{\alpha_{n0},\alpha_{n1},.....,\alpha_{A_i}\}$$

$$\phi a \circ \rightarrow y_n(\alpha ni) = 0$$

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y_o

$$b_n = 2 \int_c^c \frac{f(x)-f(-x)}{2} \sin \frac{N\eta x}{c} dx = \frac{1}{c} \left[\int_o^c f(x) \sin \frac{N\eta x}{c} dx - \int_o^c f(x) \sin \frac{N\eta x}{c} dx \right]$$

$$x=-x \qquad \qquad \qquad dx=-dx$$

$$\int_o^c f(x) \sin \frac{N\eta x}{c} (-dx)$$

$$f(x) = \frac{1}{2\pi} \sum_{n=1}^{\infty} b_n \sin nx \quad (-c, c)$$

$$f(x) = h(x) + g(x) \quad \text{and} \quad g(-x) = -g(x) \quad , \quad h(-x) = h(x)$$

$$h(x) = \frac{f(x) + f(-x)}{2} \quad g(x) = \frac{f(x) - f(-x)}{2}$$

$$h(x) = \frac{1}{2} [f(x) + f(-x)]$$

$$h(x) = \frac{1}{2} [f(x) + f(-x)]$$

$$h(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\eta x}{c} \quad a_n = \frac{2}{c} \int_0^c h(x) \cos \frac{n\eta x}{c} dx$$

$$f(x) = h(x) + g(x) \quad (-c, c)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\eta x}{c} + \sum_{n=1}^{\infty} b_n \sin \frac{n\eta x}{c} \quad \text{for } x \in (-c, c)$$

$$f(x) = \frac{1}{2} [f(x) + f(-x)]$$

$$a_n = \frac{2}{c} \int_0^c \frac{f(x) + f(-x)}{2} \cos \frac{n\eta x}{c} dx = \frac{1}{c} \left[\int_0^c f(x) \cos \frac{n\eta x}{c} dx + \int_0^c f(-x) \cos \frac{n\eta x}{c} dx \right]$$

$$\beta(z)$$

$$\int_B \beta(z) dz = 0$$

$$\int_C \frac{dz}{z^2(z^2 + q)} = \phi$$

$$\int \beta(z) dz$$

$$\int_C \beta(z) dz = \int_a^b \beta(z) dz = F(z(b)) - F(z(a))$$

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$$Ic \int_C Z^2 dZ$$

$$\int_C Pdx + Qdy = \int_R \int \left(\frac{\partial Q}{\partial X} - \frac{\partial R}{\partial y} \right) dx dy$$

$$\int_C \beta(Z) d\delta = \int_C N dx - V dy + i \int X dx + u dy$$

$$= \iint_R \left(\frac{-\partial u}{\partial y} - \frac{\partial n}{\partial x} \right) dx dy + i \iint_R \left(\frac{\partial u}{\partial y} - \frac{\partial x}{\partial y} \right) dx dy$$

$$u_x = v_y \qquad \beta(z) \qquad \mathbf{C}$$

$$Xy = -V_x$$

$$\beta(\mathbb{Z}) \qquad \mathbb{C} \qquad \beta(\mathbb{Z})$$

C

$$\omega(t) = X(t) + iy(t)$$

$$\int_a^b \omega(t) dt + \int_a^b X(t) dt + o \int_a^b y(t) dt$$

:

$$\int \theta^{i2t} dt = \sqrt{3}/2 + i/4$$

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$$\operatorname{Re} \int_a^b \omega(t) dt = \int_a^b \operatorname{Re} |Z(t)| dt$$

$$\int \mathbb{Z} \omega(t) dt = \mathbb{Z} \int_a^b \omega(t)$$

$$r_0 e^{\theta_0} = \int \omega(t) dt \Rightarrow \int (e^{-u\theta_0} \omega(t)) dt \leq r_0 c \int \operatorname{Re}(e^{-1\theta_0} \omega(t)) dt !$$

$$\left| \int_a^b X(t) dt \right| \leq \int_a^b |\omega(t)| dt$$

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$$Z$$

$$\int_a^b \beta(Z) dt = \int_{Z_1}^{Z_1} (u/x(t) + iv(x(t), y(t)))(dX' + 1dy') dt$$

$$= \int_{Z/t_0}^{Z/t_1} (Ux' - Vy') + i(uy' + Vx') dt$$

$$= \int_{t_0}^{t_1} (Ux' + Vy')dt + \int_{t_0}^{t_1} (Uy' + Vx')dt$$

$$\int_{-C} \beta(Z) dZ - \int_C \beta(Z) dZ$$

$$(1-x^2)y'' - 2xy' + y = 0$$

$$y_1 = a_0 + \sum_{h=1}^{\infty} a_2 h \times 2h$$

$$y_2 = a_1 x + \sum_{h=1}^{\infty} a_{2h+1} \times 2h + 1$$

$$\lambda = n(n+1) \qquad P_n(x) \qquad n$$

$$P_n(x) - \frac{1}{2^n} \sum \frac{(-1)^x (2n-2h)!}{h!(n-2h)h!(n+x)} X^{n-2h} \quad n = 0, 1, \dots$$

$$P_0(x)=1$$

$$P_1(x)=X$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x)=\frac{1}{2}(3S-3)$$

$$P_4\left(x \right)=\frac{1}{2}\left(3\,s\times 4\,-\,30\,x^2\,+\,3\right)$$

$$\begin{array}{l} P_n(-x)=(-1)P_n(x) \\ (i-x^s)y^n-3\times y'+n(n-1)y=0 \end{array} \quad !$$

$$\int\limits_{-1}^1P_m\left(x\right)+P_n\left(x\right)d\times=\int mn\qquad m\neq n$$

$$\phi_n(x)=\frac{P_n(x)}{|P_n(x)|}\rightarrow\{\phi_n(x)\}$$

$$P_n(x)=\frac{1}{2^n}\frac{1}{n!}\frac{d^n}{d\times n}(x^2-1)^n$$

$$P_{n+1}(x)-xp_n=\frac{n}{2^nn!}D^{n-1}u^n\,,\quad u=(x^2-1)$$

$$P_{n+1}'(x)-P_n'(x)=(2n+1)P_n(x)$$

$$|P_n(x)|=\sqrt{\frac{2}{2n+1}}\rightarrow \phi_n(x)=\sqrt{\frac{2n+1}{2}}P_n(x)$$

$$\beta(x)=\int\limits_{n=0}^{\infty}A_nP_n(x)\Rightarrow A_n=\frac{2n+1}{2}\int\limits_{-1}^1\beta(x)P_n(x)dx$$

$$r\frac{\partial^2}{\partial r^2}(ru)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\bigg(\sin\theta\frac{\partial v}{\partial\theta}\bigg)=0\qquad 0<r<c\,\,\,,\,\,\,0<\vartheta<\pi$$

$$V(c,\partial)=F(\theta)$$

$$V\left(r,\theta\right)=\sum_{n=0}^{\infty}A_n\left(\frac{r}{c}\right)^nP_n\left(\cos\theta\right)$$

$$=1/c=\left[\int\limits_o^cf(x)Cos\frac{n\eta x}{c}dx+\int\limits_{-c}^of(x)Cos\frac{n\eta x}{c}dx\right]=1/c\int\limits_{-c}^cf(x)Cos\frac{n\eta x}{c}dx$$

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